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This method of geometrizing is, as Judge Chase rightly says, a kind of mental gymnastics, but in principle it is in no wise different from Euclidean geometry and solid geometry. It is a construction of pure thought and is the product of generalizing the idea of dimension which creates new possibilities, incompatible though they may be with actual space.

There are some who talk about the possibility that our objective space may not be Euclidean but like one of the hyperspaces of modern mathematicians. But the hypothesis is worthless if we consider that objective space does not consist of objective things, but is a mere scope of motion. Whatever may be said in justification and in praise of metageometry, one thing is sure: There is no mathematician who for the sake of calculating distances, loci, or angles in the conditions of actual space would utilize or seriously recommend the use of any other but the Euclidean geometry.

## SPACE OF FOUR DIMENSIONS.

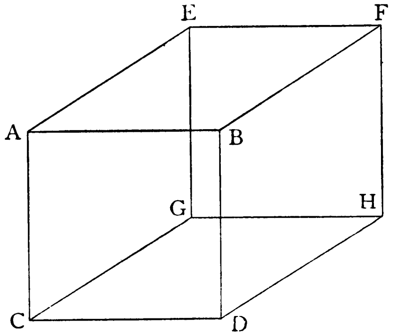
The several conceptions of space of more than three dimensions are of a purely abstract nature, yet they are by no means vague, but definitely determined by the conditions of their construction. Therefore we can determine their abstract thought and very details with perfect exactness and formulate in abstract thought the laws of four-, five-, six-, and $n$-dimensional space. The difficulty with which we are beset in constructing $n$-dimensional spaces consists in our inability to make them representable to our senses. Here we are confronted with what may be called the limitations of our mental constitution. These limitations, if such they be, are conditioned by the nature of our mode of motion, which if reduced to a mathematical system needs for a description in definite terms three coordinates, and this means that our space-conception is tridimensional.

We ourselves are tridimensional; we can measure the space in which we move with three co-ordinates, yet we can definitely say that if space were four-dimensional, a body constructed of two factors, so as to have a four-dimensional solidity, would be expressed in the formula:

$$(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4.$$

We can calculate, compute, excogitate, and describe all the characteristics of four-dimensional space, so long as we remain in the realm of abstract thought and do not venture to make use of our motility and execute our plan in an actualized construction of motion. From the standpoint of pure logic, there is nothing irrational about the assumption; but as soon as we make an *a priori* construction of the scope of our motility, we find out the incompatibility of the whole scheme.

In order to make the idea of a space of more than three dimensions plausible or intelligible, we resort to the relation between two-dimensional beings and tridimensional space. The nature of tridimensional space may be indicated yet not fully represented in two-dimensional space. If we construct a square upon the line AB one inch long, it will be bounded by four lines each one an inch in length. In order to construct upon the square ABCD a cube of the same measure, we must raise the square by one inch into the third dimension in a direction at right angles to its surface, the result being a figure bounded by six surfaces, each of which is a one-inch square. If two-dimensional beings who could not rise into the third dimension wished to gain an idea of space of a higher dimensionality and picture in their own two-dimensional mathematics the results of three dimensions, they might push out the square in any direction within their own plane to a distance of one inch, and then connect all the corners of the image of the square in its new position with the corresponding points of the old square. The result would be what is to us tridimensional beings the picture of a cube.



When we count the plane quadrilateral figures produced by this combination, we find that there are six corresponding to the boundaries of a cube. We must bear in mind that only the original and the new square will be real squares, the four intermediary figures which have originated incidentally through our construction of moving the square to a distance, exhibit a slant which our two-dimensional beings must regard as distortions of a rectangular relation, which faultiness has been caused by the insufficiency of their methods of representation. Moreover all squares count in full, and where their surfaces overlap they count double.

Two-dimensional beings having made such a construction must however bear in mind that the field covered by the sides, GEFH and BFDH, does not take up any room in their own plane, for it is only a picture of the extension which reaches out either above or below their own plane; and if they venture out on this field covered by their construction, they have to remember that it is as empty and unoccupied as the space beyond the boundaries AC and AB.

Now if we tridimensional beings wish to do the same, how shall we proceed?

We must move a tridimensional body in a rectangular direction into a new (i. e., the fourth) dimension, and, being unable to accomplish this, we may represent the operation by mirrors. Having three dimensions we need three mirrors standing at right angles. We know by *a priori* considerations according to the principle of our construction that the boundaries of a four-dimensional body must be solids, i. e., tridimensional bodies, and while the sides of a cube (algebraically represented by $a^3$) must be six surfaces (i. e., two-dimensional figures, one at each end of the dimensional line) the boundaries of an analogous four-dimensional body, built up like the cube and the square on a rectangular plan, must be eight solids, i. e., cubes. If we build up three mirrors at right angles and place any object in the intersecting corner we shall see the object not once but eight times. The body is reflected below, and the object thus doubled is mirrored not only on both upright sides but also in addition in the corner beyond, appearing in either of the upright mirrors coincidingly in the same place. Thus the total multiplication of our tridimensional boundaries of a four-dimensional complex is rendered eightfold.

We must now bear in mind that this representation of a fourth dimension suffers from all the faults of the analogous figure of a cube in two-dimensional space. The several figures are not eight independent bodies but they are mere boundaries and the four dimensional space is conditioned by their interrelation. It is that unrepresentable something which they enclose, or in other words, of which they are assumed to be boundaries. If we were four-dimensional beings we could naturally and easily enter into the mirrored space and transfer tri-dimensional bodies or part of them into those other objects reflected here in the mirrors representing the boundaries of four-dimensional objects. While thus on the one hand the mirrored pictures would be as real as the original object, they would not take up the space of our three dimensions, and in this respect our method of representing the fourth dimension by mirrors would be quite analogous to the cube pictured on a plane surface, for the space to which we (being limited by our tridimensional space-conception) would naturally relegate the seven additional mirrored images is unoccupied and, if we make the trial, will be found empty.

Further experimenting in this line would render constructions of a more complicated character more and more difficult although not quite impossible. Thus we might represent the formula $(a+b)^4$ by placing a wire model of a cube, representing the proportions $(a+b)^3$ in the corner of our three mirrors, and we would then verify by ocular inspection the truth of the formula

$$a^4+4a^3b+6a^2b^2+4ab^3+b^4.$$

However we must bear in mind that all the solids here seen are merely the boundaries of four dimensional bodies. All of them with the exception of the ones in the inner corner are scattered around, and yet the analogous figures would have to be regarded as being most intimately interconnected, each set of them forming one four-dimensional complex. Their separation is in appearance only, being due to insufficiency of our method of presentation.

We might obviate this fault by parceling our wire cube and instead of using three large mirrors for reflecting the entire cube at once, we might insert in its dividing planes double mirrors, i. e., mirrors which would reflect on the one side the magnitude $a$ and on the other the magnitude $b$. In this way we would come somewhat nearer to a faithful representation of the nature of four-dimensional space, but the model, being divided up into a number of mirror-walled rooms, would become extremely complicated, and it would be difficult for us to bear always in mind that the mirrored spaces count on both sides at once, although they overlap and (tridimensionally considered) seem to fall the one into the other, thus presenting to our eyes a real labyrinth of spaces that exist within each other without interfering with one another. They thus render new depths visible in all three dimensions, and in order to represent the whole scheme of a four-dimensional complex in its full completeness, we ought to have three mirrors at right angles placed at every point in our tridimensional space.

Our scheme of inserting mirrors at every point is impossible, but the idea will render the nature of four-dimensional space approximately clear. If we were four-dimensional beings we would be possessed of the mirror eye which in every direction could look straightway into every corner of the third dimension. This seems incredible, but it cannot be denied that tridimensional space lies open to an inspection from the domain of the fourth dimension, just as every point of a Euclidean plane is open to inspection from above to tridimensional vision. Of course we may demur (as we actually do) to believing in the reality of a space of four dimensions, but that being granted, the inferences can not be doubted. EDITOR.

## PROF. LAWRENCE H. MILLS OF OXFORD.

Among those appointed by the Hebdomadal Council of Oxford to represent the University at the coming Congress of Orientalists at Copenhagen the veteran Professor Mills is prominent. He has lately reached his seventy-first birthday, and, though he has long been in a painful condition from various complications of disease, his general health is fairly good, and his professional activities are very extensive. He has now strong hopes of living to complete his various works. *The Dictionary of the Gāthic Language of the Zend Avesta* is approaching its three hundredth page, bringing up the Gāthas to about 977 pages in all. The next section will soon be delivered to purchasers. He will also be shortly delivering the first chapter of the Yasna in the four texts: Avesta, Pahlavi, Sanskrit and Persian, with translations of the first three as in the Gāthas (Brockhaus), but he is forced to abandon the favored synoptical form of that earlier book on account of the Pahlavi text which is given both in the original character from the *Muséon* of Louvain, Belgium, with all the variants of all the manuscripts, also in the original character, as well as in the transcription of it from the *Zeitschrift* of the German Oriental Society, with the translation from the *Journal of the Royal Asiatic Society.* On account of this twofold edition of the Pahlavi this one chapter will occupy at least one hundred pages in large form. Mr. Mills has edited in all some fifty odd chapters of the Yasna, now deciphered with the collation of all of the manuscripts, but, save in the Gāthas and in Yasna, the first variants are not as yet fully published, though they are in manuscript. About thirty-two of these chapters have appeared in the *Zeitschrift* of the German Oriental Society, with translations of some thirty-four in the *Journal of R. A. S.*, London.

This work being the crux of Avesta philology, requires reconstructions of the Avesta texts, as of their translations at every step. No lexicography is to be compared with it in difficulty, for